# CALCULATION OF THE STANDARD DEVIATION OF THE STANDARDISED MEAN IN KIWIQC

**If we had independent (non-correlated) random standardised variates, then the standard deviation of the standardised mean ($z_m$)would be given by**

$$\sigma_{\bar{z}} = \frac{1}{\sqrt{n}} \qquad \textbf{[1]}$$

**where *n* is the number of levels of QC. This assumes that the means and SDs of the individual QC levels are calculated from a large number of batches (>100).**

**Example. For three QC levels (*n* = 3) the "6 sigma" control limits are**

$$\pm 3\sigma_{\bar{z}} \quad = \quad \frac{3}{\sqrt{3}} \quad = \quad \pm 1.732$$

**However, this calculation is not appropriate for multiple QC levels, as they are usually correlated.**

**For non-independent (correlated) random standardised variates:**

$$\sigma_{\bar{z}} \approx \frac{\left(1+(n-1)\bar{r}\right)^{0.5}}{\sqrt{n}} \qquad \textbf{[2]}$$

**where** $\bar{r} = (r_{12} + r_{13} + r_{23})/3$ **and *r* is the correlation coefficient. This is the expression for the standard deviation of the standardised mean ($z_m$) used in KiwiQC**

**Example: for three QC levels (ie *n* = 3) where say** $\bar{r} = 0.5$**, the "6 sigma" control limits are**

$$\pm 3\sigma_{\bar{z}} \approx \frac{3\sqrt{\left(1+(3-1)\times 0.5\right)}}{\sqrt{3}} = \pm 2.449$$

**Derivation of equation 2.**

Variance of the mean ($\sigma_{\bar{x}}^2$) = expectation of the mean squared ($E(\bar{x})^2$).

$$E(\bar{x})^2 = E(x_1 + x_2 + ... + x_n)(x_1 + x_2 + ... + x_n)/n^2$$
$$= E(x_1^2 + x_2^2 + ... + x_n^2 + 2x_1x_2 + 2x_1x_3 + ... + 2x_{n-1}x_n)/n^2$$

Now $E(x_i) = \sigma_i^2$ and $E(x_i x_j) = \sigma_{ij}$ where $\sigma_i^2$ is the variance of $x_i$ and $\sigma_{ij}$ is the covariance of $x_i$ and $x_j$.

In KiwiQC, the QC values are standardised (mean = 0, sd = 1) so that all $\sigma_i^2 = 1$, and $\sigma_{ij} = r_{ij}$ where *r* is the correlation coefficient. If it is assumed that all the $r_{ij}$ are equal, and using *z* to indicate standardised values, we have

$$E(\bar{z})^2 = \sigma_{\bar{z}}^2 = \left(n + 2r_{ij}n(n-1)/2\right)/n^2$$

Simplifying gives

$$\sigma_{\bar{z}}^2 = \left(1 + (n-1)r_{ij}\right)/n$$

In fact the $r_{ij}$ are not all equal in the QC situation, so as an approximation we replace $r_{ij}$ in the formula by the $\bar{r}$, the mean of the $n(n-1)/2$ pair-wise correlations between the *n* levels of QC.
This gives

$$\sigma_{\bar{z}} \approx \left(1 + (n-1)\bar{r}\right)^{0.5} / \sqrt{n}$$

Hence approximate ±3sd control limits are given by

$$\pm 3\left(1 + (n-1)\bar{r}\right)^{0.5} / \sqrt{n}$$

**KiwiQC web page**
http://www.cdhb.govt.nz/chlabs/endo/KiwiQcWebPage.htm